information about myself. Hence, any hesitation I may have concerning "the issue of whether or not I infer" may be due to self-doubts arising from the possibility that I am deceiving myself about either the ground of the inference or what is inferred from that ground. It cannot be correct, therefore, to claim that "there is no such thing as evidence for saying 'I infer.'" In some cases there is.

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NOTES

[1] D. G. Brown, "The Nature of Inference," *Philosophical Review*, July 1955, p. 354.
[2] *Ibid.*
[3] *Ibid.*, p. 352.
[4] For a contrary view see *loc. cit.*, p. 352.
[5] *Op. cit.*, p. 354.

Three-Valued Logic

by HILARY PUTNAM
PRINCETON UNIVERSITY

LET us make up a logic in which there are three truth-values, T, F, and "M," instead of the two truth-values T and F. And, instead of the usual rules, let us adopt the following:

a. If either component in a disjunction is true ("T"), the disjunction is true; if both components are false, the disjunction is false ("F"); and in all other cases (both components middle, or one component middle and one false) the disjunction is middle ("M").

b. If either component in a conjunction is false ("F"), the conjunction is false; if both components are true, the conjunction is true ("T"); and in all other cases (both components middle, or one component middle and one true) the conjunction is middle ("M").

c. A conditional with true antecedent has the same truth-value as its consequent; one with false consequent has the same truth-value as the denial of its antecedent; one with true consequent or false antecedent is true; and one with both components middle ("M") is true.

d. The denial of a true statement is false; of a false one true; of a middle one middle.

These rules are consistent with all the usual rules with respect to the values T and F. But someone who accepts *three* truth values, and who

accepts a notion of tautology based on a system of truth-rules like that just outlined, will end up with a different stock of tautologies than someone who reckons with just two truth values.

Many philosophers will, however, want to ask: *what could the interpretation of a third truth-value possibly be?* The aim of this paper will be to investigate this question. It will be argued that the words "true" and "false" have a certain "core" meaning which is *independent* of *tertium non datur*, and which is capable of precise delineation.

I

To begin with, let us suppose that the word 'true' retains at least this much of its usual force: if one ever says of a (tenseless) statement that it is true, then one is committed to saying that it was always true and will always be true in the future. For example, if I say that the statement 'Columbus crosses [1] the ocean blue in fourteen hundred and ninety-two' is true, then I am committed to the view that it *was* true, e.g., in 1300, and will be true in 5000 (A.D.). Thus 'true' cannot be identified with *verified*, for a statement may be verified at one time and not at another. But if a statement is *ever* accepted as verified, then at that time it must be said to have been true also at times when it was not verified.

Similarly with 'false' and 'middle'; we will suppose that if a statement is ever called 'false,' then it is also said never to have been true or middle; and if a statement is ever said to be middle, it will be asserted that it was middle even at times when it may have been incorrectly called 'true' or 'false.' In other words, we suppose that 'true' and 'false' have, as they ordinarily do have, a *tenseless* character; and that 'middle' shares this characteristic with the usual truth-values.

This still does not tell one the "cash value" of calling a statement 'middle.' But it does determine a portion of the syntax of 'middle,' as well as telling one that the words 'true' and 'false' retain a certain specified part of *their* usual syntax. To give these words more content, we may suppose also that, as is usually the case, statements that are accepted [2] as verified are called 'true,' and statements that are rejected, that is whose denials are accepted, are called 'false.' This does not determine that any particular statements must be called 'middle'; and, indeed, someone could maintain that there are some statements which have the truth-value middle, or some statements which could have the truth-value middle, without ever specifying that any particular statement has this truth-value. But certain limitations have now been imposed on the use of the word 'middle.'

In particular, statements I call 'middle' must be ones I do not accept or reject at the present time. However, it is not the case that 'middle' *means*

"neither verified nor falsified at the present time." As we have seen, 'verified' and 'falsified' are *epistemic* predicates—that is to say, they are relative to the *evidence* at a particular time—whereas 'middle,' like 'true' and 'false' is not relative to the evidence. It makes sense to say that 'Columbus crosses the ocean blue in fourteen hundred and ninety-two' was verified in 1600 and not verified in 1300, but not that it was true in 1600 and false in 1300.

Thus 'middle' cannot be *defined* in terms of 'verified,' 'falsified,' etc. What difference does it make, then, if we say that some statements—in particular some statements not now known to be true or known to be false—may *not* be either true or false because they are, in fact, middle? The effect is simply this: that one will, as remarked above, end up with a different stock of tautologies than the usual.

Someone who accepts the three-valued logic we have just described will accept a disjunction when he accepts either component, and he will reject it when he rejects both components. Similarly, he will accept a conjunction when he accepts both components, and he will reject it when he rejects either component. This is to say that the behavior of the man who uses the particular three-valued logic we have outlined is not distinguishable from the behavior of the man who uses the classical two-valued logic in cases wherein they know the truth or falsity of all the components of the particular molecular sentences they are considering.

However, they will behave differently when they deal with molecular sentences some of whose components have an unknown truth-value.[3] If it is known that snow is white, then the sentence 'snow is white $\vee \sim$ snow is white' will be *accepted* whether one uses classical two-valued logic or the particular three-valued logic we have described. But if one does not know whether or not there are mountains on the other side of the moon, then one will *accept* the sentence 'there are mountains on the other side of the moon $\vee \sim$ there are mountains on the other side of the moon' if one uses the classical two-valued logic, but one will say 'I don't know whether that's true or not' if one uses three-valued logic, or certain other nonstandard logics, e.g., "Intuitionist" logic.[4]

II

At this point the objection may be raised: "But then does this notion of a 'middle' truth-value make sense? If having a middle truth-value does not mean having what is ordinarily called an *unknown* truth value; if, indeed, you can't tell us *what* it does mean, then does it make sense at all?"

Analytic philosophers today normally reject the demand that concepts be translatable into some kind of "basic" vocabulary in order to be meaningful. Yet philosophers often reject the possibility of a three-valued logic

(except, of course, as a mere formal scheme, devoid of interesting interpretations), just on the ground that no satisfactory *translation* can be offered for the notion of having a "middle" truth-value. Indeed, if the notion of being a statement with a middle truth-value is defined explicitly in terms of a two-valued logic or metalogic, then one usually obtains a *trivial* interpretation of three-valued logic.

Does a middle truth-value, within the context of a system of three-valued logic of the kind we have described, have a use? The answer is that it does, or rather that it belongs to a *system* of uses. In other words, to use three-valued logic makes sense in the following way: to use a three-valued logic means to adopt a different way of using logical words. More exactly, it corresponds to the *ordinary* way in the case of molecular sentences in which the truth-value of all the components is known (i.e., we "two-valued" speakers say it is known); but a man reveals that he is using three-valued logic and not the ordinary two-valued logic (or partially reveals this) by the way he handles sentences which contain components whose truth-value is not known.

There is one way of using logical words which constitutes the ordinary two-valued logic. If we are using three-valued logic,[5] we will behave in exactly the same way except that we will employ the three-valued rules and the three-valued definition of 'tautology.' Thus 'using three-valued logic' means adopting a systematic way of using the logical words which agrees in certain respects with the usual way of using them, but which also disagrees in certain cases, in particular the cases in which truth-values are unknown.

III

Of course one might say: "Granted that there is a consistent and complete way of using logical words that might be described as 'employing a three-valued logic.' But this alternative way of using logical words—alternative to the usual way—doesn't have any *point*."

And perhaps this is what is meant when it is said that three-valued logic does not constitute a real alternative to the standard variety: it exists as a calculus, and perhaps as a nonstandard way of using logical words, but there is no *point* to this use. This objection, however, cannot impress anyone who recalls the manner in which non-Euclidean geometries were first regarded as absurd; later as mere mathematical games; and are today accepted as portions of fully interpreted physical hypotheses. In exactly the same way, three-valued logic and other nonstandard logics had first to be shown to exist as consistent formal structures; secondly, uses have been found for some of them—it is clear that the Intuitionist school in mathe-

matics, for example, *is*, in fact, systematically using logical words in a nonstandard way, and it has just been pointed out here that one might use logical words in still another nonstandard way, corresponding to three-valued logic (that is, that this would be a form of linguistic behavior reasonably represented by the formal structure called 'three-valued logic'). The only remaining question is whether one can describe a physical situation in which this use of logical words would have a point.

Such a physical situation (in the microcosm) has indeed been described by Reichenbach.[6] And we can imagine worlds such that even in *macrocosmic* experience it would be physically impossible to either verify or falsify certain empirical statements. For example, if we have verified (by using a speedometer) that the velocity of a motor car is such and such, it might be impossible in such a world to verify or falsify certain statements concerning its position at that moment. If we *know* by reference to a physical law together with certain observational data that a statement as to the position of a motor car can *never* be falsified or verified, then there may be some point to not regarding the statement as true or false, but regarding it as "middle." It is only because, in macrocosmic experience, everything that we regard as an empirically meaningful statement seems to be at least potentially verifiable or falsifiable that we prefer the convention according to which we say that every such statement is either true or false, but in many cases we don't know which.

Moreover, as Reichenbach shows, adopting a three-valued logic permits one to preserve both the laws of quantum mechanics and the principle that no causal signal travels with infinite speed—"no action at a distance." On the other hand, the laws of quantum mechanics are logically incompatible with this principle if ordinary two-valued logic is used.[7] This inconsistency is not usually noticed, because in quantum mechanics no causal signal is ever *detected* traveling faster than light. Nevertheless it can be shown—as Einstein and others have also remarked [8]—that the mathematics of quantum mechanics entails that in certain situations a causal signal *must have* traveled faster than light.

A working physicist can dismiss this as "just an anomaly"—and go on to accept *both* quantum mechanics and the "no action" principle. But a logician cannot have so cheerful an attitude toward logical inconsistency. And the suggestion advanced by Bohr, that one should classify the trouble-making sentences as "meaningless" (complementarity) involves its own complications. Thus the suggestion of using a three-valued logic makes sense in this case, as a move in the direction of simplifying a whole system of laws.

To return to the macrocosmic case (i.e., the "speedometer" example),

Bohr's proposal amounts to saying that a syntactically well-formed sentence (e.g., 'my car is between 30 and 31 miles from New York') is in certain cases *meaningless* (depending on whether or not one uses a speedometer). Reichenbach's suggestion amounts to saying that it is meaningful, but neither true nor false (hence, "middle"). There seems little doubt that it would be simpler in practice to adopt Reichenbach's suggestion. And I suspect that beings living in a world of the kind we have been describing would, in fact, regard such statements as *neither true nor false*, even if no consideration of preserving simple physical laws ("no action at a distance") happened to be involved. This "suspicion" is based on two considerations: (a) The sentences admittedly have a very clear cognitive use; hence it is unnatural to regard them as "meaningless." (b) There is no reason why, in such a world, one should even consider adopting the rule that every statement is either true or false.

On the other hand, in our world (or in any world in which Planck's constant *h* has a small value) it would be very unnatural to adopt three-valued logic for describing ordinary macrocosmic situations. Suppose we did. Then there would be two possibilities: (a) We maintain that certain sentences are "middle," but we never say which ones. This seems disturbingly "metaphysical." (b) We say that some particular sentence S is middle.

This last course is, however, fraught with danger. For, although "S is middle" does not *mean* "S will never be either verified or falsified," it *entails* "S will never be either verified or falsified." And the prediction that a particular sentence will never be either verified or falsified is a strong empirical prediction (attention is confined to synthetic sentences for the sake of simplicity); and one that is itself always potentially falsifiable in a world where no physical law prohibits the verification of the sentence S, regardless of what measurements may have antecedently been made.

Thus, the reason that it is safe to use three-valued logic in the Reichenbachian world (the microcosm) but not in the "actual" world (the macrocosm) is simply that in the Reichenbachian world one can, and in the "actual" world one cannot, *know in advance* that a particular sentence will never be verified or falsified. It is not that in a "Reichenbachian" world one *must* call sentences that will never be verified or falsified "middle," but, rather, that in *any* world *only* (but not necessarily *all*) such sentences must be classified as "middle." This follows from the fact that sentences that are said to be verified are also said to be true; sentences that are said to be falsified are also said to be false; and the truth values are "tenseless." Thus it would be a contradiction to say that a sentence is middle, but may someday be verified.

These features of the use of "true" and "false" seem indeed to be con-

stitutive of the meaning of these words. *Tertium non datur* might also be said to be "true from the meaning of the words 'true' and 'false' "—but it would then have to be added that these words have a certain core meaning that can be preserved even if *tertium non datur* is given up. One can abandon two-valued logic without changing the meaning of 'true' and 'false' *in a silly way*.

IV

Analytic philosophers—both in the "constructivist" camp and in the camp that studies "the ordinary use of words"—are disturbingly unanimous in regarding two-valued logic as having a privileged position: privileged, not just in the sense of corresponding to the way we *do* speak, but in the sense of having no serious rival for *logical* reasons. If the foregoing analysis is correct, this is a prejudice of the same kind as the famous prejudice in favor of a privileged status for Euclidean geometry (a prejudice that survives in the tendency to cite "space has three dimensions" as some kind of "necessary" truth). One can go over from a two-valued to a three-valued logic without *totally* changing the meaning of 'true' and 'false'; and not just in *silly* ways, like the ones usually cited (e.g., equating truth with high probability, falsity with low probability, and middlehood with "in between" probability).

Indeed, so many strange things have been said about two- and three-valued logic by philosophic analysts who are otherwise of the first rank that it would be hopeless to attempt to discuss them all in one short paper. But two of these deserve special mention:

a. It has often been said that "even if one uses a three-valued object language, one must use two-valued logic in the metalanguage." In the light of the foregoing, this can hardly be regarded as a *necessary* state of affairs. Three-valued logic corresponds to a certain way of speaking; there is no difficulty in speaking in that way about any particular subject matter. In particular, one may assign truth-values to molecular sentences in the way we have discussed, whether one is talking about rabbits or languages or metalanguages.

(Of course, if one is *explaining* three-valued logic to someone who only uses two-valued logic one will employ a two-valued language as a medium of communication. This is like remarking that one uses French to teach Latin to French schoolboys.)

b. It has been argued [9] that the meaning of 'true' has been made clear by Tarski for the usual two-valued system, but that no analogous clarification is available for 'true' in three-valued systems. The obvious reply is that the famous biconditional '*snow is white*' *is true if and only if snow*

is white is perfectly acceptable even if one uses three-valued logic. Tarski's criterion has as a consequence that one must accept *'snow is white' is true* if one accepts *snow is white* and reject *'snow is white' is true* if one rejects *snow is white*. But these (along with the "tenseless" character of the truth-values) are just the features of the use of 'true' and 'false' that we have preserved in our three-valued logic. It is, for instance, just because *tertium non datur* is independent of these features that it is possible for Intuitionist logicians to abandon it without feeling that they are changing the "meaning" of 'true' and 'false.'

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NOTES

[1] 'Crosses' is used here "tenselessly"—i.e., in the sense of "crossed, is crossing, or will cross."

[2] More precisely, S is accepted if and only if 'S is true' is accepted.

[3] The distinction between sentences and statements will be ignored, because we have passed over to consideration of a formalized language in which it is supposed that a given sentence can be used to make only one statement.

[4] Cf. Alonzo Church's *Introduction to Mathematical Logic* (Princeton, N.J.: Princeton University Press, 1956), p. 141. Intuitionist logic is not a truth-functional logic (with any finite number of truth-values). However, the rules given above hold (except when both components are "middle" in the case of rules b and c) provided truth is identified with intuitionist "truth," falsity with "absurdity," and middlehood with being neither "true" nor "absurd."

[5] In this paper, 'three-valued logic' means the system presented at the beginning. Of course, there are other systems, some of which represent a more radical change in our way of speaking.

[6] *Philosophic Foundations of Quantum Mechanics* (University of California, 1944).

[7] *Ibid.*, pp. 29–34.

[8] A. Einstein, B. Podolsky, N. Rosen, "Can Quantum Mechanical Description of Reality Be Considered Complete?" *Physical Review*, 47:777 (1935).

[9] For example, Hempel writes in his review of the previously cited work by Reichenbach (*Journal of Symbolic Logic*, volume 10, p. 99): "But the truth-table provides a (semantical) interpretation only because the concept of *truth* and *falsity*, in terms of which it is formulated, are already understood: they *have the customary meaning* which can be stated in complete precision by means of the *semantical definition of truth*." (Italics mine.)